

MATHEMATICAL FOUNDATIONS:

LINEAR ALGEBRA, MATRICES, NUMERICAL ANALYSIS, ABSTRACT ALGEBRA, AND DIFFERENTIAL CALCULUS



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Mathematical Foundations: Linear Algebra, Matrices, Numerical Analysis, Abstract Algebra, and Differential Calculus



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Preface

Mathematics forms the structural backbone of science, engineering, economics, and emerging computational disciplines. A strong command of its foundational branches is not optional—it is essential. Mathematical Foundations: Linear Algebra, Matrices, Numerical Analysis, Abstract Algebra, and Differential Calculus is designed to provide a coherent and rigorous introduction to the core areas of mathematics that underpin modern theoretical and applied studies.

This book is written primarily for undergraduate and early postgraduate students who require clarity, logical development, and conceptual depth rather than fragmented or purely procedural learning. The text integrates five fundamental domains of mathematics that are often studied in isolation but are deeply interconnected in practice. Linear Algebra and Matrices establish the language of vector spaces and transformations essential for data science, physics, and engineering. Numerical Analysis addresses real-world computational limitations and equips readers with methods to approximate solutions where exact results are impractical. Abstract Algebra introduces the structural thinking necessary for higher mathematics, cryptography, and theoretical computer science. Differential Calculus provides the analytical tools required to model change, optimize systems, and understand continuous phenomena.

The presentation emphasizes precise definitions, step-by-step derivations, and carefully selected examples to strengthen mathematical reasoning. Proofs are included where conceptually necessary, while computational techniques are explained with sufficient depth to build problem-solving confidence. Each chapter is structured to move from fundamental principles to applications, ensuring a logical progression of ideas.

This book does not attempt to oversimplify mathematics. Instead, it respects the intellectual demands of the subject and the learner. Readers who engage seriously with the material will develop not only technical competence but also the analytical discipline required for advanced study and research. It is hoped that this text will serve as a reliable foundation and a reference point throughout the reader's academic journey in mathematics and related fields.

Acknowledgement

We express our sincere gratitude to all those who have contributed, directly or indirectly, to the completion of this book, Mathematical Foundations: Linear Algebra, Matrices, Numerical Analysis, Abstract Algebra, and Differential Calculus. This work is the result of sustained academic effort, teaching experience, and a commitment to presenting fundamental mathematical concepts with clarity, logical structure, and rigor.

We are deeply thankful to our mentors, senior academicians, and colleagues for their valuable guidance, constructive criticism, and insightful discussions, which played a significant role in shaping the content and improving the academic quality of this manuscript. Their suggestions helped refine explanations, strengthen theoretical arguments, and enhance the overall coherence of the text.

We also acknowledge the contribution of our students, whose classroom interactions, questions, and problem-solving approaches continually motivated us to revisit and clarify complex ideas. Their engagement influenced the pedagogical framework of this book and ensured that the material remains accessible without compromising mathematical depth.

We gratefully acknowledge the support of our respective institutions for providing an academic environment conducive to teaching, research, and scholarly writing. Access to institutional libraries, reference materials, and academic resources was essential in the preparation of this work. We also thank the editorial and publishing team for their professionalism, careful review, and dedication to maintaining high academic standards throughout the publication process.

Finally, we express our heartfelt appreciation to our families for their patience, encouragement, and understanding during the course of this work. Their support made it possible to devote sustained time and effort to the completion of this book. We hope that this text will serve as a valuable resource for students, educators, and researchers in mathematics.

**Jagadeesha K. C.
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About the Authors



Jagadeesha.K.C. is an accomplished mathematician with M.Sc. and M.Phil degrees in Mathematics. He has dedicated sixteen years to teaching in various Government Degree Colleges across Karnataka. His extensive experience has shaped the mathematical foundation of countless undergraduate students. Currently, he is pursuing his Ph.D. in Mathematics at Davangere University, further deepening his academic expertise. As an Associate Professor at Government First Grade College (GFGC), Tumkur, he continues to inspire students through his engaging teaching methods. His research interests reflect a deep commitment to advancing mathematical knowledge. He has published several research papers in Scopus-indexed journals. These publications highlight his scholarly contributions to the field. His dedication to both teaching and research exemplifies his passion for mathematics. Through his work, he strives to contribute meaningfully to the academic community and the broader landscape of mathematical education.



Siddalingaswamy R. holds an M.Sc. and B.Ed. from Tumkur University and is presently pursuing a Ph.D. in Mathematics. He qualified the K-SET examination for Lectureship in 2013. Currently, he serves as an Assistant Professor of Mathematics at Government First Grade College, B.H. Road, Tumkur. With 11 years of teaching experience at the degree level, he has contributed extensively to academic development and student mentorship. He has served as a Board of Examiners (BOE) member for both Tumkur University and Davangere University. His academic interests span various domains of mathematics, and he remains actively involved in research and departmental activities. Through his teaching and scholarly engagement, he continues to

inspire students and contribute to the field of higher education.



M. S. Sunitha is an Associate Professor of Mathematics with an M.Sc and M.Phil, and is currently pursuing her Ph.D. at GITAM University. She has been serving in the Department of Higher Education for the past 15 years, contributing significantly to undergraduate mathematics education. Presently, she is working at the Government First Grade College, B.H. Road, Tumakuru. Her academic journey reflects a deep commitment to teaching, research, and curriculum development. She has actively served as a BOE and BOS member in Bangalore University and various autonomous colleges. Her leadership skills are well recognized, having served as the Chairperson of the Board of Examiners at Tumkur University for the academic year 2024–2025. She is passionate about mentoring students and inspiring them toward mathematical thinking. Her areas of interest span pure and applied mathematics, with a focus on innovative pedagogical practices. She consistently participates in academic conferences, workshops, and faculty development programmes to enhance her scholarly expertise. As a researcher, she is keen on contributing impactful publications to the field of mathematics. Her dedication to academic excellence continues to guide her professional pursuits and future goals.



Nagaraja N. K. is an Assistant Professor of Mathematics with qualifications including M.Sc., B.Ed., and KSET. He is currently serving at Smt. & Sri Y. E. Rangaiah Shetty Government First Grade College, Challakere Road, Pavagada, in Tumkur District. With eight years of experience in teaching at the degree college level, he has significantly contributed to undergraduate mathematics education. He has also served as a Board of Examiners (BOE) member at Davangere University during the academic year 2022–23. In addition, he was a BOE member at Tumkur University for the academic year

2023–24. Prior to his current role, he worked as a Lecturer in Government PU College, Tadas, Shiggaon Taluk, Haveri District. His service at the PU level spanned four years, from 2013 to 2017. Through these roles, he has gained rich teaching and academic administrative experience. His professional journey reflects dedication to both pedagogy and academic growth. He continues to contribute actively to the field of mathematics education through his teaching, evaluation work, and institutional involvement.

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ABOUT THE BOOK:

Mathematical Foundations: Linear Algebra, Matrices, Numerical Analysis, Abstract Algebra, and Differential Calculus presents a concise and rigorous treatment of core mathematical disciplines essential for higher studies in science, engineering, and applied fields. The book integrates linear algebra and matrices for understanding vector spaces and linear systems, numerical analysis for approximation methods and error estimation, abstract algebra for the study of algebraic structures such as groups, rings, and fields, and differential calculus for analyzing limits, derivatives, and change. Each topic is developed with clear definitions, logical progression, and illustrative examples, ensuring conceptual clarity without compromising mathematical rigor. Designed primarily for undergraduate and early postgraduate students, the book also serves as a dependable reference for educators seeking a structured and academically sound resource in foundational mathematics.



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